RTSA terminology
What is real time

Real-time digital signal processing (DSP):

The term “real time” is derived from early work on digital simulations of physical systems. A digital system *simulation* is said to operate in *real time* if its operating speed matches that of the *real system* which it is simulating.

To analyze signals in real time means that the analysis operations must be performed fast enough to accurately process all signal components in the frequency band of interest.   This definition implies that we must:

1. Sample the **input** signal fast enough to satisfy Nyquist criteria. This means that the sampling frequency must ***exceed*** twice the bandwidth of interest.
2. Perform all computations continuously at a fast enough rate that the output analysis keeps up with the changes in the input signal.

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Two important needs for Real Time Spectrum Analyzers

- **Processing all information contained in a signal in real time requires:**
  - **Enough capture bandwidth to support the signal of interest.**
  - **High enough ADC clock rate to exceed the Nyquist criteria for the capture bandwidth.**
  - **Long enough capture interval to support the narrowest resolution bandwidth (RBW) of interest.**
  - **High enough DFT transform rate to exceed the Nyquist criteria for the RBW of interest.**
  - **Overlapping DFT frames**
    - The amount of overlap depends on the window function
    - The Window function is determined by the RBW

- **Discovering, and Capturing transient events requires:**
  - **Minimum event duration for 100% probability of capturing a single non-repetitive event.**
    - A minimum event is defined as the narrowest rectangular pulse that can be captured with certainty.
  - **Enough capture bandwidth to support the signal of interest.**
  - **High enough ADC clock rate to exceed the Nyquist criteria for the capture bandwidth.**
  - **Long enough capture interval to support the narrowest resolution bandwidth (RBW) of interest.**
  - **High enough DFT transform rate to support the minimum event duration.**

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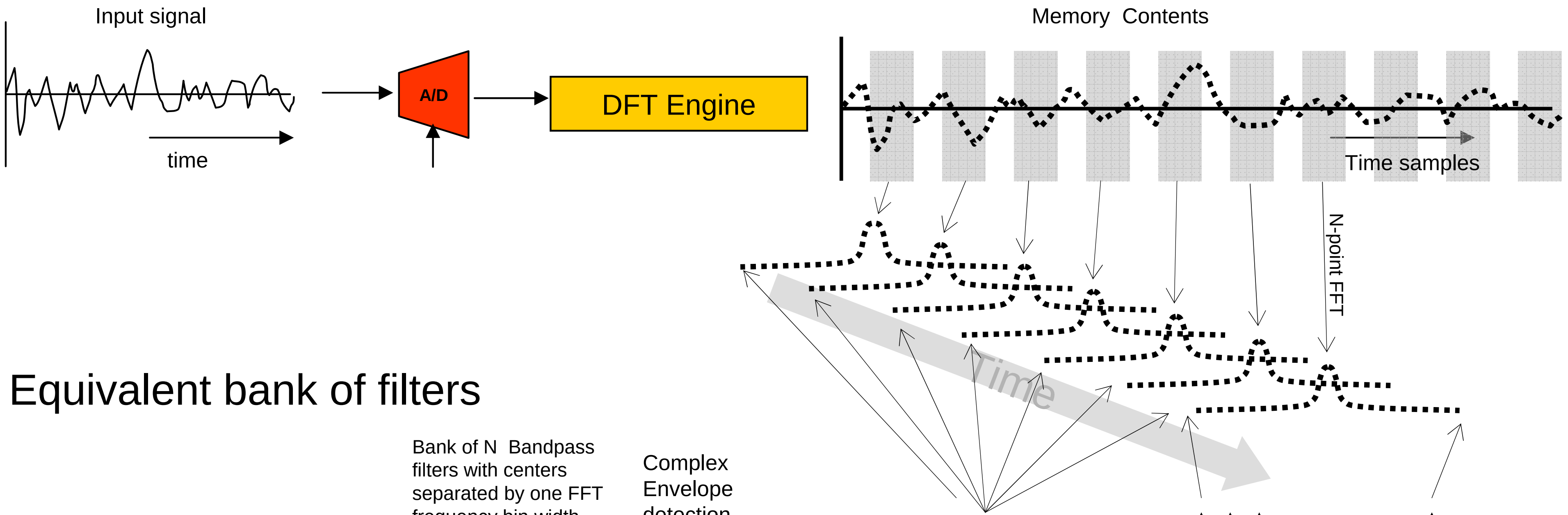
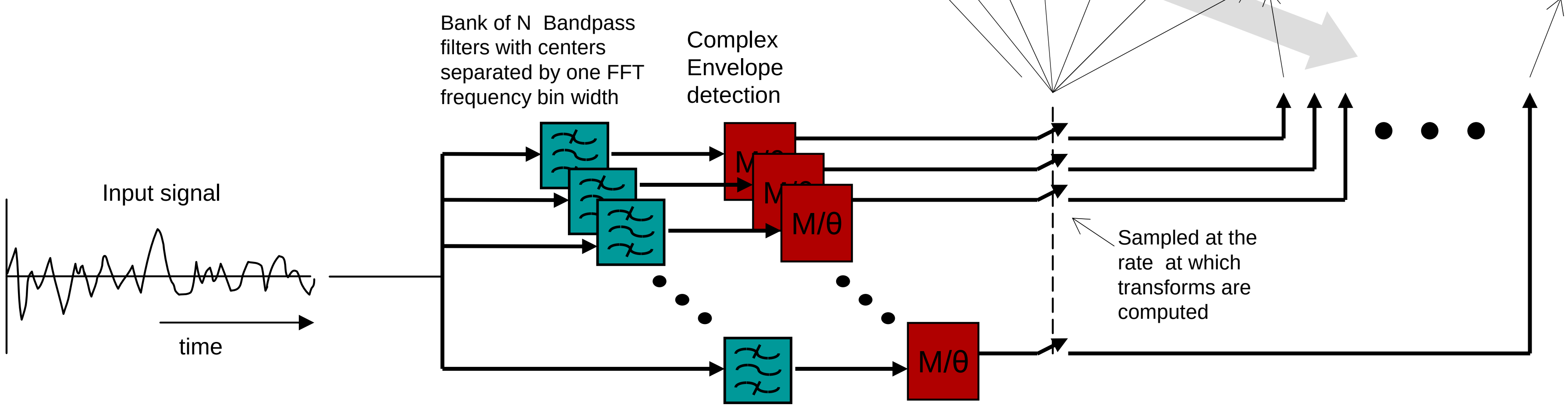
Performing repetitive Discrete Fourier Transforms is equivalent to passing signals through a bank-of-filters

DFT* Based Spectrum Analysis



Equivalent bank of filters



* The Fast Fourier Transform (FFT) is a common implementation of a Discrete Fourier Transform (DFT).

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Real-time spectrum analysis

- Discrete Fourier transforms
  - Spectrum analysis, also called Fourier analysis, requires that signals be observed in the frequency domain. When using DSP, this implies performing discrete Fourier transforms (DFTs) on time sampled data.
- Real time Fourier analysis using DSP
  - Refer to diagram on the previous slide
  - Discrete Fourier transforms are continuously performed on the sampled input signal.
  - This is equivalent to passing the signal through a bank of bandpass filters, each of which has the bandwidth and separation of the DFT bins. The complex envelope (I and Q or Magnitude and Phase) is computed for each frequency domain bin of the DFT output each time a new DFT is performed.
  - **Criteria for Real Time Spectrum analysis**
    1. **The input signal must be sampled fast enough to meet the Nyquist criteria for the bandwidth of interest.**
    2. **The DFT computations must be performed fast enough such that the Nyquist criteria is met for each of the DFT bins.**
       - **It can be shown that this is equivalent to having no gaps between DFT frames.**
       - **Windowing and other practical implementations require DFT frames to overlap.**

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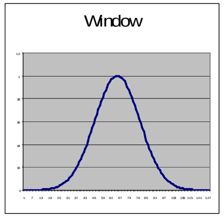
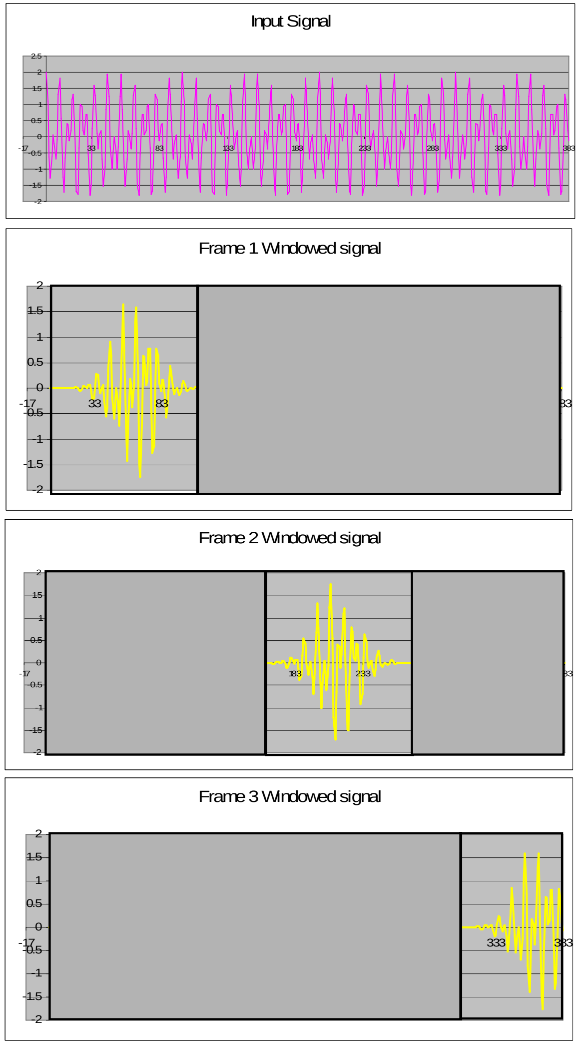
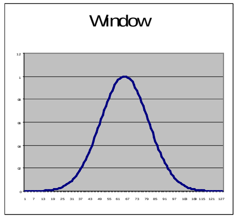
Windowing and DFT frame overlap

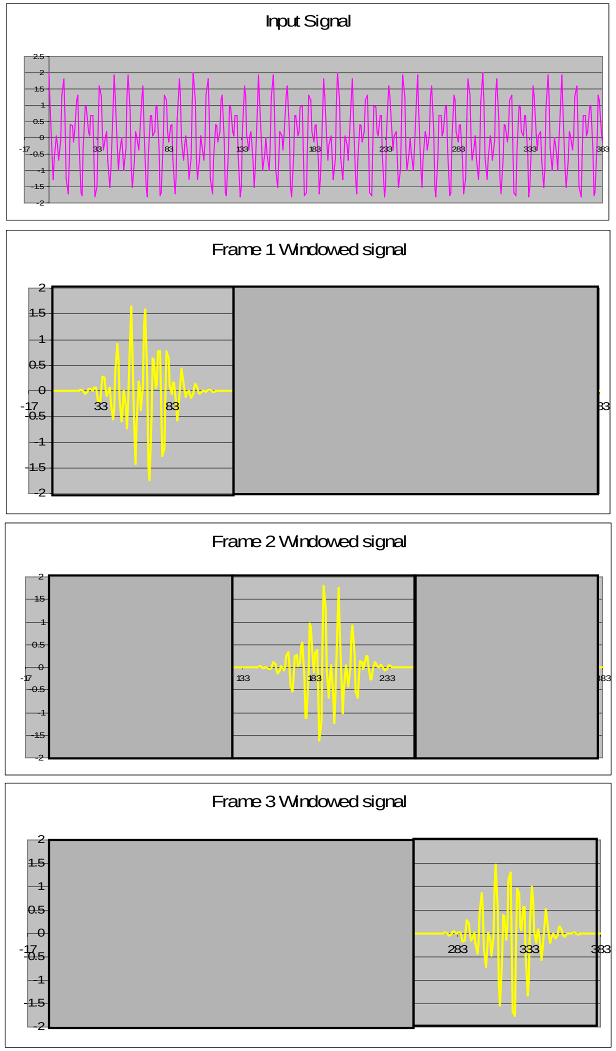


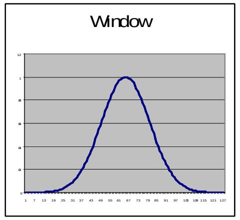
Processing gap between DFT frames



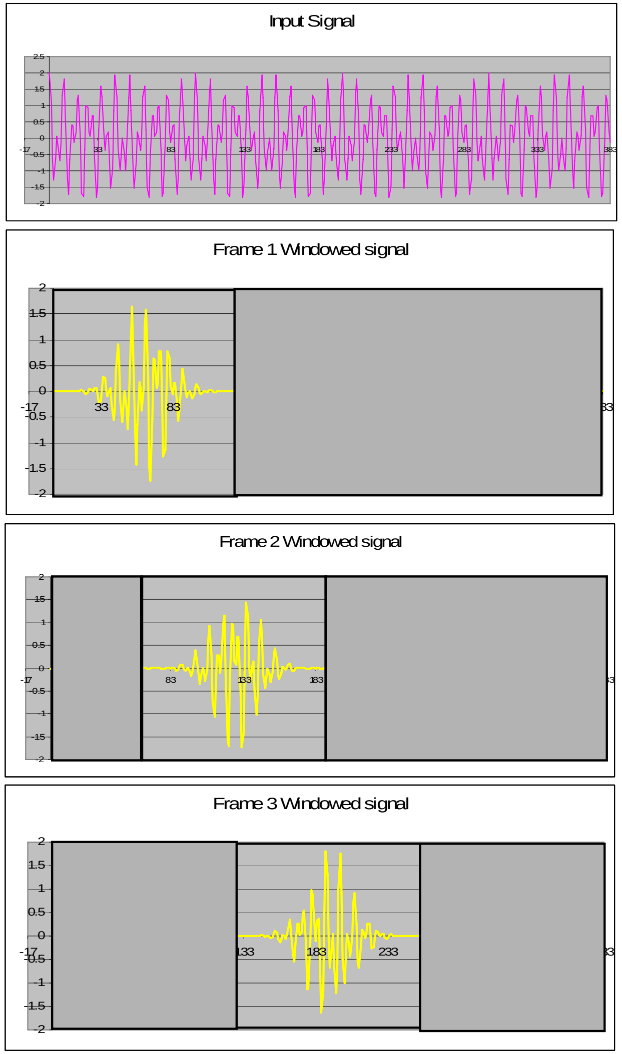


DFT Frames end-to-end with no gap

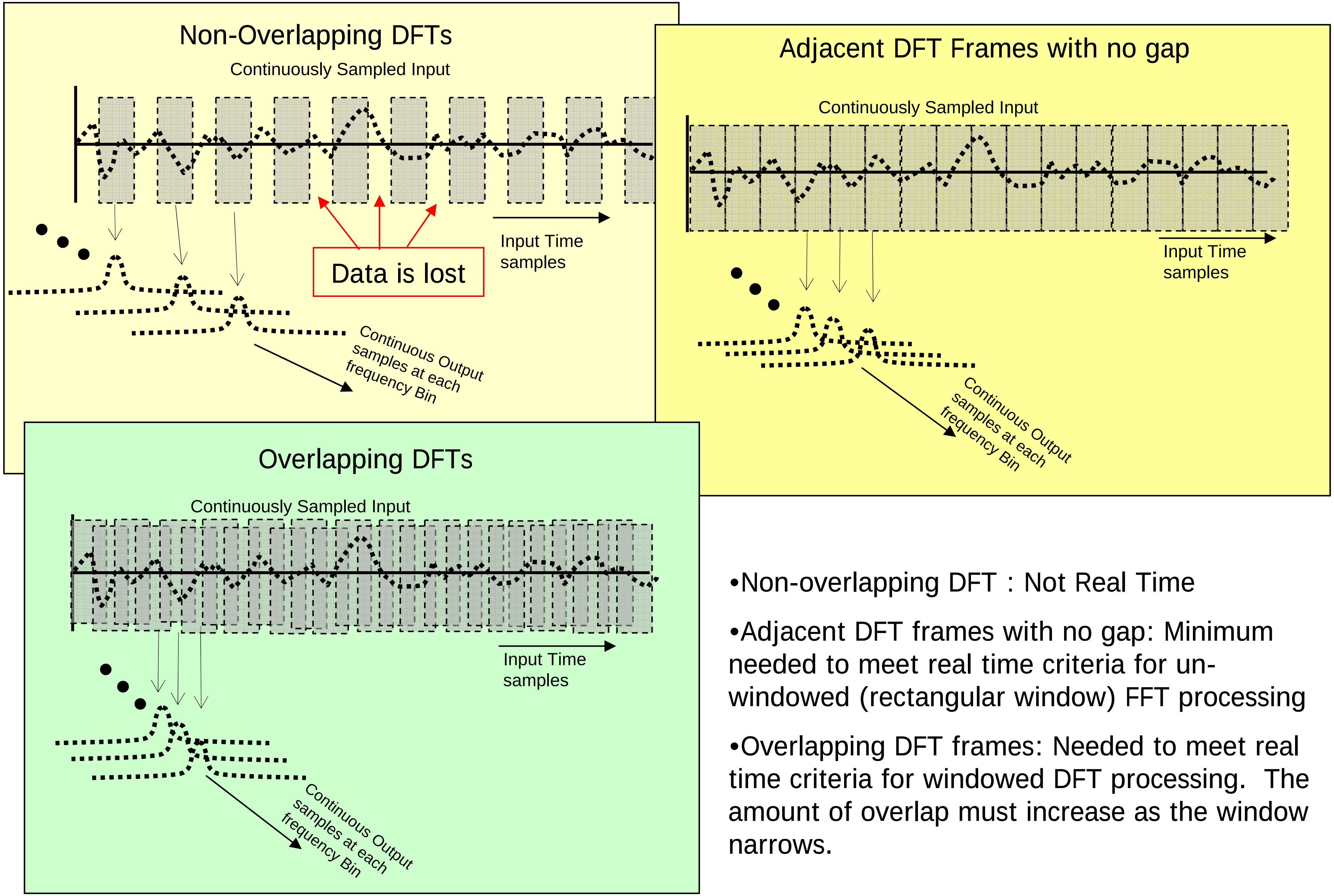




Overlapping DFT Frames



Implications of overlapping DFT frames



- Non-overlapping DFT : Not Real Time
- Adjacent DFT frames with no gap: Minimum needed to meet real time criteria for un-windowed (rectangular window) FFT processing
- Overlapping DFT frames: Needed to meet real time criteria for windowed DFT processing. The amount of overlap must increase as the window narrows.

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Minimum Event Duration for 100% probability of discovery or capture at full amplitude

Overlapped DFT frames

$$T_{min} = 2\,T_{acq} - T_{ol}$$

Gap between frames

$$T_{min} = 2\,T_{acq} + T_{gap}$$

Where:

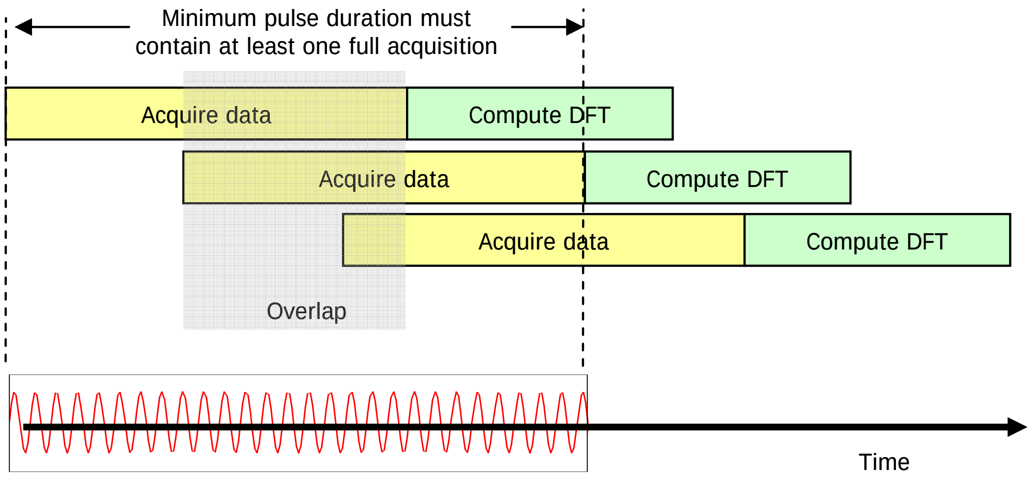
T_{min}=Minimum time for 100% probability if intercept

T_{ol}=Overlap time

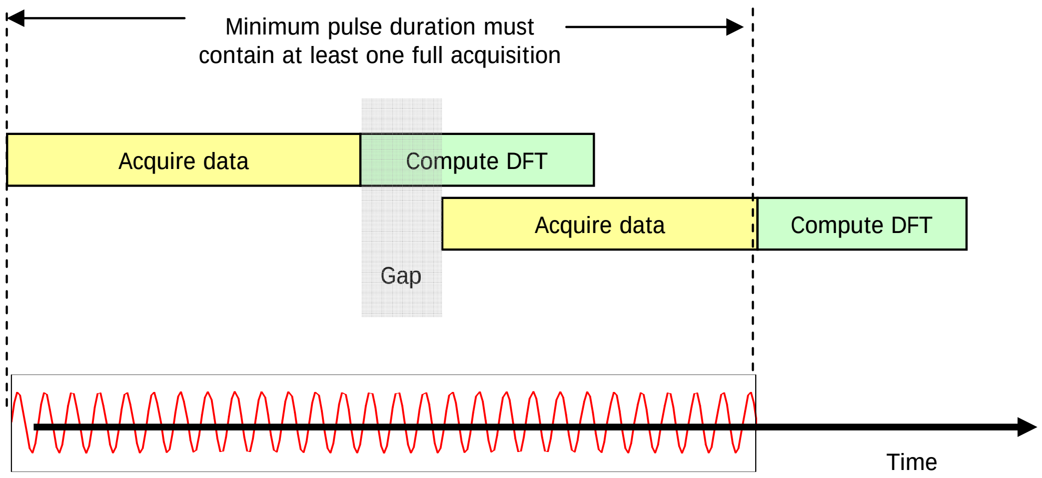
T_{gap}=Gap between acquisitions when there is no overlap

T_{acq}=Time to acquire a DFT frame

Overlapped DFT frames



Gap between DFT frames



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Minimum Event Duration for 100% Probability of Intercept and DFT overlap.

Model	Max Acquisition Bandwidth	DPX		Frequency Mask Trigger		Power/Level Trigger
		Min Pulse Duration for 100% probability of discovery	DFT overlap	Min Pulse duration for 100% probability of trigger	DFT overlap	Minimum pulse duration for 100% probability of trigger at widest BW
RSA6000	40 MHz	31 µSec	≧50 % for spans ≦10 MHz	30.1 µSec	≧50 %	20 nSec
RSA6000 Opt 110	110 MHz	24 µSec	≧50 % for spans ≦ 10 MHz	10.24 µSec	≧50 %	6.67 nSec
RSA 3408B	36 MHz	31 µSec	≧50 % for Span ≦ 10 MHz	20 µSec	≧50 %	20 nSec
RSA 3300B	15 MHz	41 µSec	≧50 % for spans ≦ 10 MHz	30 µSec	≧50 %	40 nSec

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Appendix

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Sampling rate vs DFT rate in RTSAs

Meeting Nyquist criteria for sampled systems

- Real-valued input signal **input**:
  - The rate at which time domain **samples** are taken is at least twice the bandwidth of the signal of interest.
- Complex valued signal **input** (I and Q):
  - The rate at which I and Q are each sampled must be at least equal to the bandwidth of the signal of interest. Each complex input is effectively two samples.
- Complex valued **output**
  - *Complex output* is the general case for DFT computations. Either the Cartesian (I and Q) or the polar (magnitude and phase) are commonly used.
  - The rate at which **new DFTs** are computed is at least equal to the bandwidth of each output bin. Each complex output for each DFT bin is effectively two samples.

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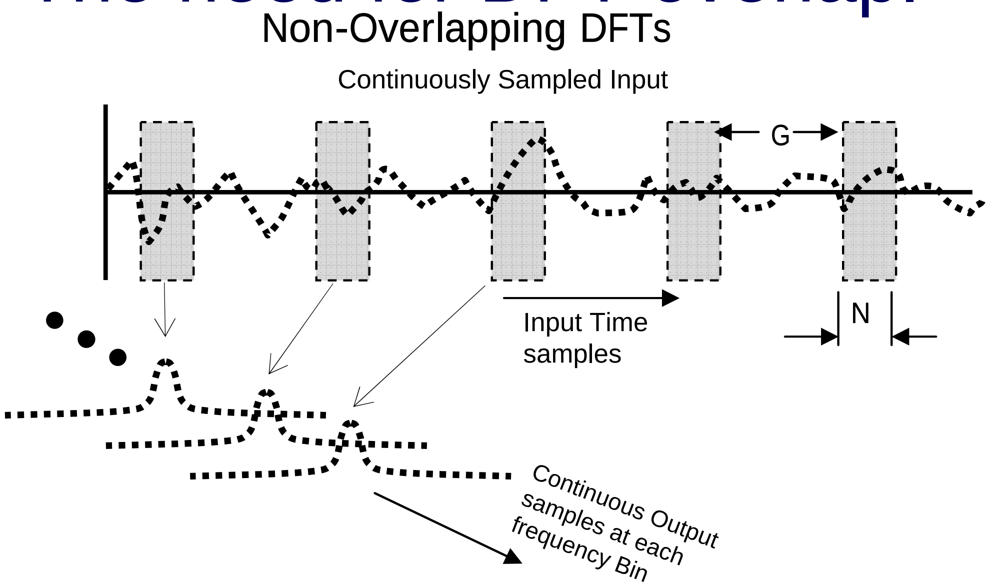
The need for DFT overlap.



N= number of points in the FFT

G= number of samples in the gap

F_S=Sampling frequency

R=Transform rate

BW= output bin width

$BW=F_S/N$

$R=F_S/(N+G)$

$R<BW$ for $G>0$

- Meeting the Nyquist criteria for complex valued samples requires that the transform rate, R be at least equal to the bin width, BW.
- There must be overlap

The effect of Windowing

- The mathematics of finite length Discrete Fourier transforms have the inherent assumption that the signal is periodic, with a period equal to the length of the transform.
- There can be edge effects that cause spectral leakage when the time-domain signal in the DFT does not have this exact periodicity.
- Windowing functions are used to de-emphasize the samples in the edges of a DFT frame. The window function is multiplied with the input samples on a sample-by-sample basis.

Some definitions from a Google search

- Real-time systems are defined as those systems in which the correctness of the system depends not only on the logical result of computations, but also on the time at which the results are produced.
  - **Real-Time Systems**
    - The International Journal of Time-Critical Computing Systems
    - Editor-in-Chief: Tarek F. Abdelzaher; Giorgio Buttazzo; Krithi Ramamritham
- A computer system that responds to input signals fast enough to keep an operation moving at its required speed. Examples are video game computers and videoconferencing systems as well as computers used to control airplanes, space shuttles and other "real" equipment.
  - PCMAG.com encyclopaedia

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