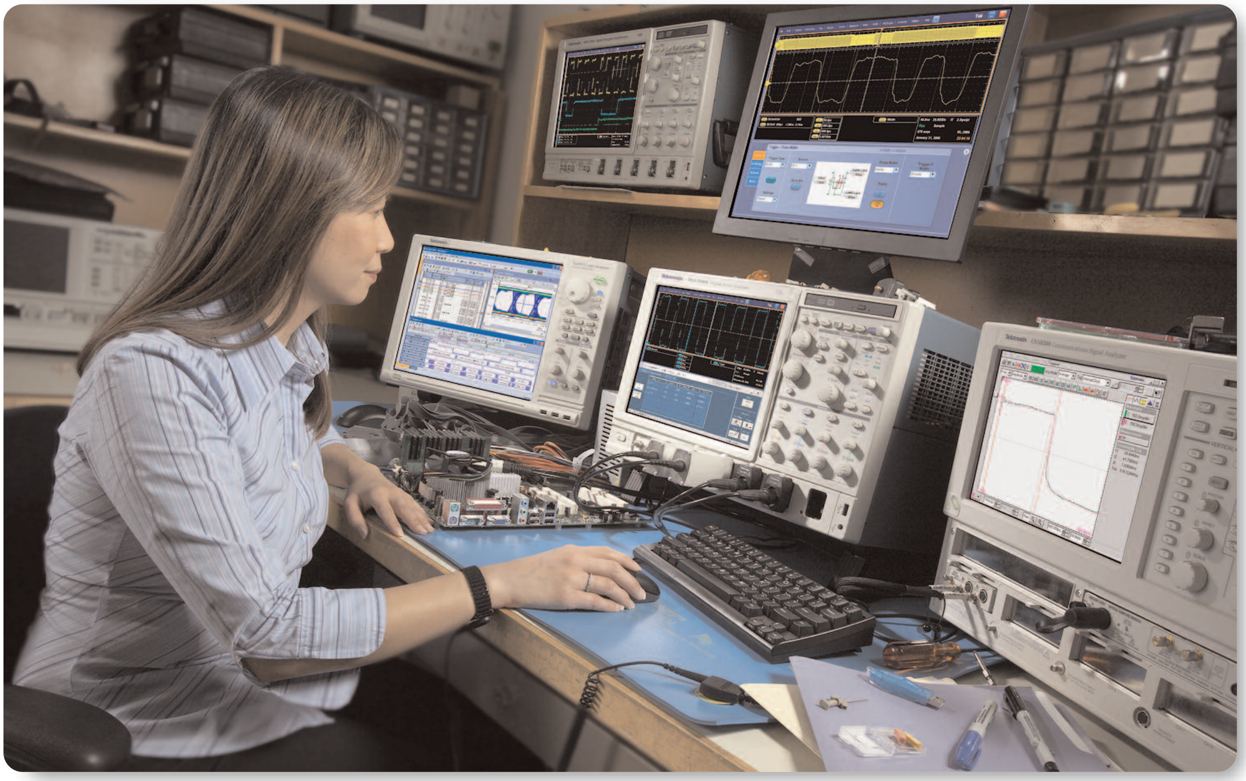
Effective Bits

Effective Bits Testing Evaluates Dynamic Performance of Digitizing Instruments



The Effective Bits Concept

Whether you are designing or buying a digitizing system, you need some means of determining actual, real-life digitizing performance. How closely does the output of any given analog-to-digital converter (ADC), waveform digitizer or digital storage oscilloscope actually follow any given analog input signal?

At the most basic level, digitizing performance would seem to be a simple matter of resolution. For the desired amplitude resolution, pick a digitizer with the requisite number of "bits" (quantizing levels). For the desired time resolution, run the digitizer at the requisite sampling rate. Those are simple enough answers. Unfortunately, they can be quite misleading, too.

While an "8-bit digitizer" might provide close to eight bits of accuracy and resolution on DC or slowly changing signals, that will not be the case for higher speed signals. Depending on the digitizing technology used and other system factors, dynamic digitizing performance can drop markedly as signal speeds increase. An 8-bit digitizer can drop to 6-bit, 4-bit, or even fewer effective bits of performance well before reaching its specified bandwidth.

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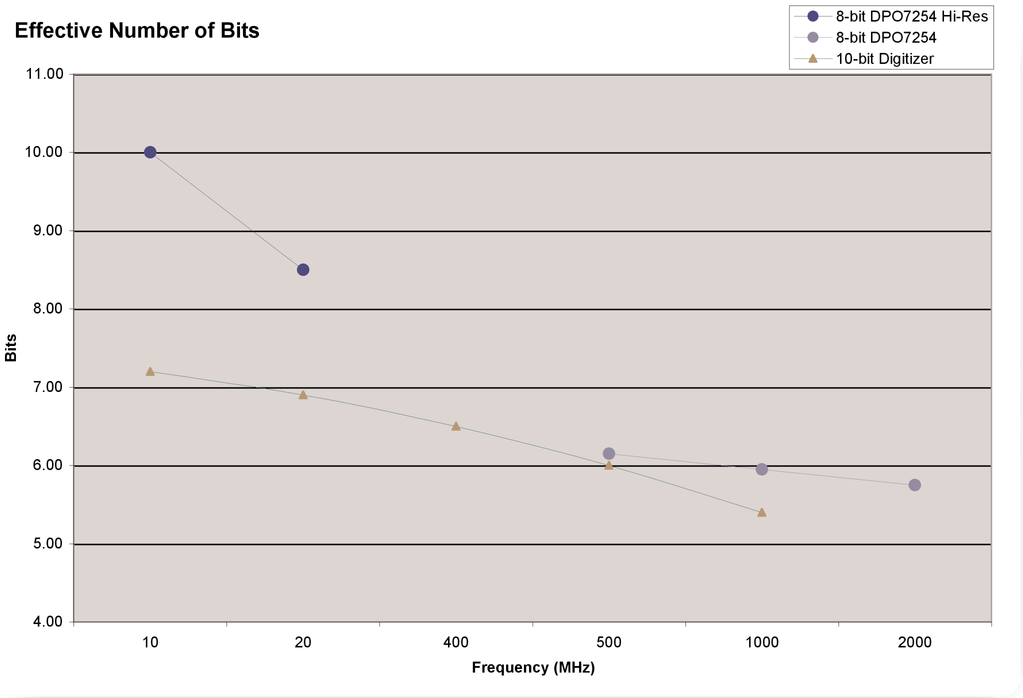


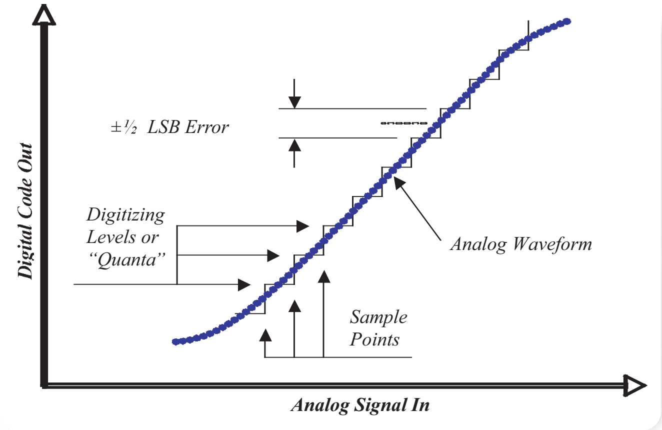
Figure 1. *When comparing digitizer performance, testing the full frequency range is important.*

If you are designing an ADC device, a digitizing instrument, or a test system, it is important to understand the various factors affecting digitizing performance and to have some means of overall performance evaluation. Effective bits testing provides a means of establishing a figure of merit for dynamic digitizing performance. Not only can effective bits be used as an evaluation tool at various design stages, beginning with ADC device design or selection, but it can also be used to provide an overall system dynamic performance specification.

For those making digitizing system purchase decisions, effective bits is an equally important evaluation tool. In some instances, effective bits may already be stated as part of the system or instrument specification. This is becoming increasingly common for waveform digitizing instruments. However, effective bits may not always be specified for individual instruments or system components. Thus, it may be necessary to do an effective bits evaluation for purposes of comparison. If equipment is to be combined into a system, an effective bits evaluation can provide an overall system figure-of-merit for dynamic digitizing system performance.

Essentially, effective bits is a means of specifying the ability of a digitizing device or instrument to represent signals of various frequencies. The basic concept is illustrated in Figure 1, which shows a plot of effective bits versus frequency.

The plot in Figure 1 shows the effective number of bits of two digitizers vs. frequency. Like gain-bandwidth or Bode plots, ENOB generally – but not always – decreases with frequency. The major difference is that the ENOB plot compares digitization precision or digital bits of accuracy rather that analog gain (or attenuation) accuracy.



▶ **Figure 2.** *Quantizing error.*

What the plot in Figure 1 tells us is that effective digitizing accuracy falls off as the frequency of the digitized signal increases. In other words, an 8-bit digitizer provides eight effective bits of accuracy only at DC and low frequencies or slow signal slopes. As the signal being digitized increases in frequency or speed, digitizing performance drops to lower and lower values of effective bits.

This decline in digitizer performance is manifested as an increasing level of noise on the digitized signal. “Noise”, here, refers to any random or pseudorandom error between the input signal and the digitized output. This noise on a digitized signal can be expressed in terms of a signal-to-noise ratio (SNR),

$$SNR = rms\_signal / rms\_error \quad \textbf{[Eq. 1.]}$$

where rms (signal) is the root-mean-square value of the digitized signal and rms (error) is the root-mean-square value of the noise error. The relationship to effective bits (EB) is given by,

$$EB = \log_2(SNR) - \tfrac{1}{2}\log_2(1.5) - \log_2(A/FS) \quad \textbf{[Eq. 2.]}$$

Resolution or Effective Bits (N)	Quantizing Levels	Signal-to-Noise Ratio in dB (6.08N+1.8dB)
4	16	26.12
6	64	38.28
8	256	50.44
10	1,024	62.60
12	4,096	74.76
14	16,384	86.92
16	65,536	99.08

▶ **Table 1.** *digitizer is ±1/2 LSB of error.*

where A is the peak-to-peak input amplitude of the digitized signal and FS is the peak-to-peak full-scale range of the digitizer’s input. Other commonly used formulations include,

$$EB = N - \log_2(rms\_error / ideal\_quantization\_error) \quad \textbf{[Eq. 3.]}$$

where N is the nominal (static) resolution of the digitizer, and,

$$EB = -\log_2(rms\_error * \sqrt{12}/FS) \quad \textbf{[Eq. 4.]}$$

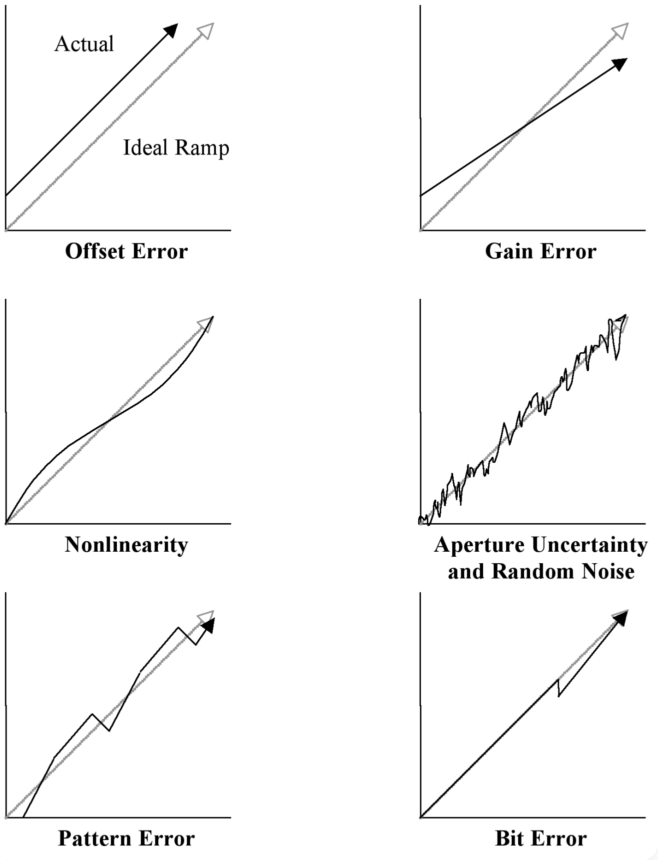
Notice that all these formulations are based on a noise, or error level, generated by the digitizing process. In the case of Equation 3, the “ideal quantization error” term is the rms error in ideal, N-bit digitizing of the input signal. Both Equations 2 and 3 are defined by the IEEE Standard for Digitizing Waveform Recorders (IEEE std. 1057). Equation 4 is an alternate form for Equation 3. It is derived by assuming that the ideal quantization error is uniformly distributed over one least significant bit (LSB) peak-to-peak. This assumption allows the ideal quantization error term to be replaced with $FS/(2^n\sqrt{12})$ where FS is the digitizer’s full-scale input range.

Another important thing to notice about these equations is that they are based on full-scale signals (FS). In actual testing, test signals at less than full scale (e.g., 50% or 90%) may be used. This can result in improved effective bits results. Consequently, any comparisons of effective bits specifications or testing must take into account test signal amplitudes as well as frequency.

Error Sources in the Digitizing Process

Noise, or error, related to digitizing can come from a variety of sources. Even in an ideal digitizer, there is a minimum noise or error level resulting from quantizing. This “quantizing error” amounts to $\pm \frac{1}{2}$ LSB (least significant bit). As illustrated in Figure 2 and Table 1, this error is an inherent part of digitizing. It is the resolution limit, or uncertainty, associated with ideal digitizing. To this basic ideal error floor, a real-life digitizer adds further errors. These additional real-life errors can be lumped into various general categories –

- DC offset (also AC offset or “pattern” errors, sometimes called “fixed pattern distortion,” associated with interleaved sampling methods)
- Gain error (DC and AC)
- Nonlinearity (analog) and Nonmonotonicity (digital)
- Phase error
- Random noise
- Frequency (time base) inaccuracy
- Aperture uncertainty (sample time jitter)
- Digital errors (e.g. data loss due to metastability, missing codes, etc.)
- And other error sources such as trigger jitter



▶ **Figure 3.** *Errors associated with non-ideal digitizing include DC offset, gain error, integral and differential non-linearity, sample jittering, and other noise contributions (stuck bits, dropped bits, etc.)*

Figure 3 illustrates some of the more basic error categories to give you a visual idea of their effects. Many of the errors encountered in digitizers are the classical error types specified or associated with any amplifier or analog network. For example, DC offset, gain error, phase error, nonlinearity and random noise can occur anywhere in the waveform capture process, from input of the analog waveform to output of digitized waveform values.

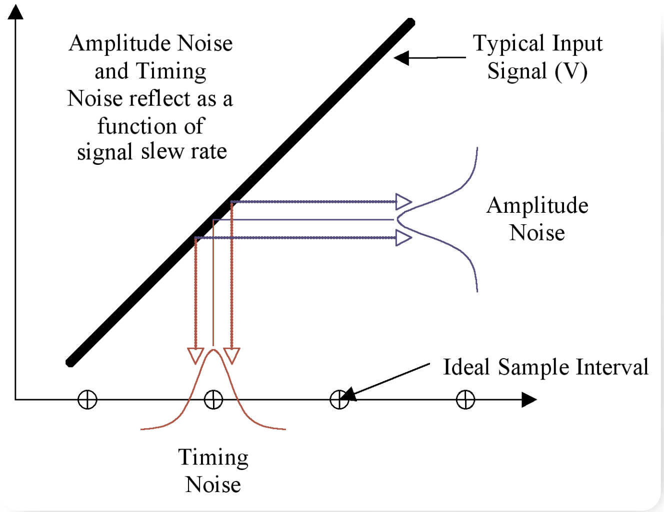
On the other hand, aperture uncertainty and time base inaccuracies are phenomena associated with the sampling process that accompanies waveform digitizing. The basic concept of aperture uncertainty is illustrated in Figure 4.

The important thing to note from Figure 4 is that aperture uncertainty results in an amplitude error and the error magnitude is slope dependant. The steeper the slope of the signal, the greater the error magnitude resulting from a time jittered sample. Aperture uncertainty is only one of many reasons for decreases in effective bits at higher signal frequencies or slopes. However, aperture uncertainty serves as a useful and graphical example for exploring input signal frequency and amplitude related issues.

To gain further insight into the effects of aperture uncertainty, consider sampling the amplitude of a sine wave at its zero crossing. For a low-frequency sine wave, the slope at the zero crossing is low, resulting in minimal error from aperture uncertainty. However, as the sine wave's frequency increases, the slope at the zero-crossing increases. The results is a greater amplitude error for the same amount of aperture uncertainty or jitter. Greater error means lower SNR and a decrease in effective bits. In other words, the digitizer's performance falls off with increasing frequency. This is expressed further by the following equation.

$$f = \frac{1}{\sqrt{6} \bullet \pi \bullet \Delta t \bullet 2^N} \quad \textbf{[Eq. 5.]}$$

In equation 5, f is the frequency of a full-scale sine wave that can be digitized to n bits with a given rms aperture uncertainty, Δt. If aperture uncertainty remains constant and frequency is increased, then the number of bits, n, must decrease in order to maintain the equality in Equation 5.



▶ **Figure 4.** *Aperture uncertainty, or sample jitter, makes an amplitude error contribution that is a function of slew rate and timing jitter. Similarly, amplitude noise can impact timing measurements.*

There is, however, a way around the necessary decrease in bits, n, for increasing frequency. This relates back to the concepts illustrated in Figure 4. If the amplitude of the sine wave is decreased from full scale, the zero-crossing slope decreases. Thus, the amplitude error decreases, resulting in a better effective bits number. This points out an important fact when comparing effective-bit numbers form various digitizers. Effective bits depends not only on frequency, but on the amplitude of the test waveform. Any one-to-one testing or comparison of digitizers must include specifications of the input waveform's amplitude (typically 50% or 90% of full scale) as well as frequency.

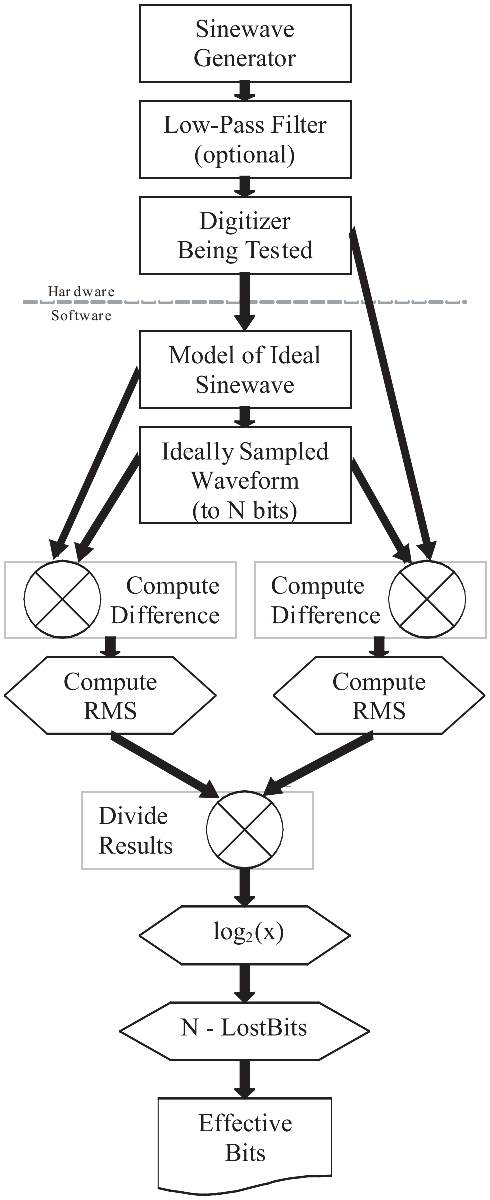
Also, it should be noted that input amplifier roll-off, post-acquisition filtering and other processing can reduce signal amplitude internal to the digitizing instrument. This can result in effective bit specifications that overstate the actual, real-life dynamic performance of the instrument.

The Effective Bits Measurement Process

Beyond the error sources and considerations mentioned thus far, there are still other possible sources of digitizing error. For example, in high-speed real-time digitizing without a sample-and-hold or track-and-hold, the least significant bits must change at extremely high rates in order to follow a fast changing signal. This puts high bandwidth requirements on the data lines and buffer inputs for these lesser bits. If these bandwidth requirements are not met, fast changing lesser bits will be "dropped", leaving the digitizer with a lower effective bits performance. This is, of course, in addition to the many other possible error sources prior to and after the digitizing device.

Rather than trying to distinguish and measure each individual error source within a digitizing system, it is easier to measure overall performance. In other words, given an ideal input signal, what are the overall error contributions of the digitizing system in the output signal? A good place to start is determining the digitizing system's SNR and the resulting effective bits as defined by Equations 2, 3 or 4. This provides an easily understood and universal figure of merit for comparisons.

The basic test process is illustrated in Figure 5. It involves applying a known, high-quality signal to the digitizer and then computer analyzing the digitized waveform. A sine wave is used as the test signal because high-quality sine waves are relatively easy to generate and characterize. The general test requirements are that the sine wave generator's performance must significantly exceed that of the digitizer under test. Otherwise, the test will not be able to distinguish digitizing errors from signal source errors. It may be necessary to add filters to the source in order to reduce source



▶ **Figure 5.** *The process of effective bits measurements.*

harmonics to levels significantly below what might be expected from the digitizer under test. To obtain an effective bits number, a perfect (idealized) sine wave is computed and fitted to the digitized sine wave. This perfect sine wave is described by,

$$A \bullet \sin(2\pi ft + \Theta) + C \qquad \text{[Eq. 6.]}$$

where A is the sine wave's amplitude, f is its frequency, Θ is phase, t is time and C is DC offset. The actual process of fitting this sine wave can use any of several software algorithm variations designed to converge quickly on an optimum result. This result is considered to be a description of the analog input to the digitizer. It should be noted, however, that because the analog signal parameters are computed from the digitizer's output, DC offset, gain, phase and frequency errors are not included. These excluded errors need to be measured by separate tests, such as a histogram test or other test appropriate to the specific error of interest. After computing a model of the ideal input sine wave, further computations are done to determine ideal sampling and digitizing of the sine wave. This simulates what the N-bit digitizer under test would produce if it were an ideal N-bit digitizer. The difference between the computed ideal sine wave and the perfectly sampled and digitized version is then computed. The rms value of this provides the ideal quantization error used in Equation 3. The rms error value used in the effective bits equations (3 and 4) is obtained by subtracting the ideal sine wave from the actual digitized sine wave and finding the rms value of the result. Or as an alternative, the rms value of the signal and the rms error can be found and used to compute SNR for use in Equation 2.

The final computation (using Equations 2, 3 or 4) results in an effective bits number for the digitizer. By keeping input signal amplitude constant for various frequencies, further effective bits numbers can be computed for the subject digitizer or digitizing system. These numbers can then be plotted against frequency to obtain a digitizer performance curve such as illustrated in Figure 1.

Effective bits lumps many of the key digitizer system errors into a figure of merit that is easy to understand and use in comparisons. As noted previously, however, effective bits does depend on the input signal's percent of full-scale digitizer amplitude. A digitizer tested at less than full-scale amplitude will generally exhibit somewhat better effective bits numbers than if tested at 100% full scale. Testing at less than full scale can be justified since most digitizer inputs are set up in actual practice to keep the input signal below full scale to avoid overdriving the digitizer. Whatever the test philosophy used – full scale or partial scale – the input test signal amplitude specification should accompany the effective bits results.

Caution also needs to be exercised in selecting frequencies for developing an effective-bits plot. If the test signal frequency is harmonically related to the digitizer's sampling rate, there is a possibility of beat frequencies interfering with the test results. Consequently, it is best to ensure that the test signal is asynchronous with the digitizer's sampling.

Converter Error	Effective Bits Test	FFT Test	Spectral Avg Test	Histogram Test
Differential Non-linearity	**Yes** part of rms error	**Yes** as elevated noise floor	**Yes** as elevated noise floor	**Yes** read out numerically as %LSB
Integral Non-linearity	**Yes** part of rms error	**Yes** as harmonics of fundamental aliased to baseband	**Yes** as harmonics of fundamental aliased to baseband	**Yes**
Missing Codes	**Yes** part of rms error	**Yes** as elevated noise floor	**Yes** as elevated noise floor	**Yes** as bins with zero counts
Aperture Uncertainty	**Yes** part of rms error	**Yes** as elevated noise floor	**Yes** as elevated noise floor	**No** averaged out
Noise	**Yes** part of rms error	**Yes** as elevated noise floor	**Yes** read out numerically as elevated noise floor	**No** averaged out
Gain Error	**No**	**No**	**NA**	**Yes** shows in peak-peak distribution
Offset Error	**No**	**No**	**NA**	**Yes** read out % of FS peak-peak value

▶ **Table 2.** *Summary of A/D Converter Dynamic Performance Tests.*

Digitizer triggering is still another area that needs caution. In general, capture of the test signal should be done in a single-shot mode. This eliminates the deleterious effects of trigger jitter from the effective bits measurement and concentrates the evaluation on the digitizer itself.

However, to operate at higher bandwidths (or sweep speeds in the case of digital storage oscilloscopes), many digitizers must use repetitive triggering and equivalent time sampling to build a full sample complement over many repetitions of the input waveform. Trigger jitter and long-term drift effects can increase the noise level associated with equivalent-time digitizers. This is most often dealt with by using signal averaging to decrease noise, thus raising the effective bits of the digitizer. If signal averaging is going to be used in conjunction with effective bits testing, the amount of signal averaging used should be stated with the effective bits results.

Also, you should be aware that the built-in signal averaging used in some digitizing instruments may employ computing at a higher resolution than the digitizer itself.

For example, an 8-bit digitizing instrument may use internal 16-bit computations for signal averaging. This high-resolution averaging can make an 8-bit digitizer appear to be a 10- or 11-bit digitizer. This, of course, can result in higher effective bits testing results when averaged input signals are used. In general, it's rarely valid to compare digitizers which use signal averaging to digitizers which capture signals on a single-shot basis, unless both digitizers can be set to a common operating mode for the purposes of “fair and equal” comparisons.

Other Dynamic Performance Tests

Beyond effective bits testing, there are still other test methods that can be used to evaluate the dynamic performance of digitizers. These methods include FFT Tests, Spectral Average Test and Histogram Tests. Generally, these tests are used to augment the results obtained by effective bits testing or to obtain specific information about some particular aspect of the digitizer's performance. Table 2 provides a summary of the error measurements made by these various tests.

Briefly, FFT testing allows measurement of the digitizer's noise floor and harmonic distortion due to integral nonlinearity. This is done by simply computing the FFT of the digitized sine wave test signal. Assuming that the FFT computation is at a much higher precision than the digitized sine wave, the noise floor of the FFT result is the noise floor of the digitizer. Also, any harmonics from nonlinearities will appear in the FFT results. The amplitudes of the harmonics indicate the degree of digitizer nonlinearity, assuming minimum harmonic distortion in the source waveform. It should be noted that interpretation of results can be impacted by the type of window used on the data and whether or not the mean was removed from the data prior to FFT application.

Spectral averaging is similar to FFT testing, except that repeated acquisitions of the test waveform are transformed and converted to frequency-domain magnitudes. The computed magnitudes are then point-by-point averaged to obtain the spectral average. This provides a leaner view of the digitizer's performance, making it easier to interpret the noise floor and harmonics. However, for useful interpretation the results should be accompanied by test signal amplitude and frequency information.

Histogram testing takes a different approach in that digitized signal code density is being looked at. A pure sine wave input is digitized by the digitizer under test. The relative number of occurrences of distinct digital output codes is referred to as the code density. This is viewed as a normalized histogram showing the occurrence frequency of each code from zero to full scale. An output "zero" code density indicates a missing code and a shift in density from the ideal typically indicates a linearity error.

Whether or not any of these additional tests are used depends on the amount of error specification desired for the digitizer in question. As indicated by Figure 1 and Table 2, effective bits provides a good overall view of digitizer dynamic performance. This can be extended with additional testing to reveal more detail about specific error sources. However, effective bits still remains the most universal method of basic specification, much as bandwidth is a basic specification for amplifiers and oscilloscopes.

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